



Murdoch
UNIVERSITY

MURDOCH RESEARCH REPOSITORY

<http://researchrepository.murdoch.edu.au/6270/>

Kissane, B. (2006) Half a century of calculation in Western Australian secondary schools. Cross Section, 16 (1). pp. 1-3.

Copyright: 2006 Mathematics Association of Western Australia
It is posted here for your personal use. No further distribution is permitted.

Half a century of calculation in Western Australian secondary schools*

Barry Kissane
School of Education
Murdoch University

Ask a person in the street to describe something mathematical and chances are that you will get a description of a calculation of some sort. For many people in the general public, mathematics is still most closely associated with calculation. Indeed, the converse is also true: people see calculations and assume they are mathematical, which partly accounts for the (mistaken) view that accounting is a branch of mathematics. While mathematics teachers have long understood the considerable differences between mathematics and calculation, it is still nonetheless the case that calculation is a component of mathematics.

In this essay, one person's observations about calculation over the lifetime of the Mathematical Association of Western Australia are briefly described. The observations have been too carelessly made to be reasonably described as a history of calculation over that time, but they do at least indicate some of the noticeable shifts, as they have affected the teaching of mathematics. The overwhelming conclusion is that the means of calculation have changed enormously over this comparatively short time span, and continue to do so, even today.

According to Larry Blakers (1978), Professor of Mathematics at the University of Western Australia, the MAWA began life in 1958 as a branch of The Mathematical Association in the UK. So, at the time of writing, the MAWA is close to celebrating its first fifty years.

The good old days?

At the time of MAWA's founding, students in school were prepared for two examinations that certified their progress in mathematics. The Junior Certificate was conducted at the end of three years of secondary school, preceded by seven years of primary school – what would nowadays be described as the end of year 10. The Leaving Certificate was conducted after a further two years, at the end of secondary school. Although many students sat the Junior exams, many others also left school and began work before reaching that stage, and a comparatively small number of students remained at school to complete the Leaving Certificate.

It is interesting to examine some of the old examination papers, which give a good sense of what was expected at that time. There were two mathematics papers, one on Arithmetic and Algebra, the other on Geometry and Trigonometry. For the present purpose, only the Arithmetic and Algebra papers are considered. In the 1955 paper, students were expected to choose five arithmetic questions from the seven offered, and given 75 minutes in which to do so. The first question (for which 15 minutes would be allocated notionally) is mostly a calculation task:

- 1 (i) Find by factors the cube root of 85.184^1 and hence write down the cube root of $.085184.^2$
(ii) Multiply $48.47 \times .578.^3$

¹ The decimal points on the paper were raised, but have been given here on the horizontal line to make them more interpretable to modern readers.

² The examination paper did not use leading zeroes, as would be regarded as good practice today, so has been faithfully reproduced here.

³ Both the word 'Multiply' and the multiplication sign were stated on the paper.

Of course, this question was to be answered without any calculation aids and at that stage calculators did not exist. There is certainly some mathematical thinking in extracting cube roots by hand, although the task involved was a routine one, as was the long multiplication algorithm expected in part (ii) of the question. These days, students could very quickly find the numerical answer to each question using a calculator, but would not be finding the cube roots ‘by factors’, as demanded.

The third question on the 1955 paper would never be seen in a classroom today:

3. Divide £64 7s. 6d.⁴ into three parts in the ratio $2.125 : 3\frac{5}{8} : .6875$

The clear intention of this question is to test students’ manipulative skills, first to express each of the three parts of the ratio using the same denominator and then to undertake the divisions and multiplications concerned. Indeed, apart from (the legitimate purpose of) determining whether students could undertake arithmetic, the intention of such a question is not clear: it is difficult to imagine an everyday (or indeed any other) context for which such a calculation was important.

That is not to say that all arithmetical questions were devoid of a context, however. The fifth question of the same paper provided a context and even the idea of an average (in an age when statistics was not an identifiable part of school mathematics):

5. A dealer bought 30 sheep at an average price of 75s. each. If 8 of them cost 72s. each, and 16 of them cost 78s. 9d. each, what was the average cost of the rest?

While it is not an easy matter to construct a context in which this particular form of the question makes sense, at least there is a context to which the calculations can be related.

The examination papers a decade later are of interest, as these were used immediately prior to the introduction of decimal currency early in 1966. The structure of the exam was essentially the same, although ten minutes of Reading Time had been added. The first question in 1965 was in many respects similar to that of 1955:

1. (a) Find the prime factors of 59400 and 207515, and give (in factors) the cube root of 59400×207515 .

(b) Divide 54.6178 by .787.
Note: Absolute accuracy is required.

The admonition regarding accuracy is a curious inclusion, as by-hand methods of calculation (in this case certainly) do not lend themselves to approximate answers, so it is difficult to imagine that students would have interpreted the questions as requiring anything other than ‘absolute accuracy’. In the ‘good old days’, arithmetic was right or wrong; students were not encouraged to provide good approximations, or even to recognise how to do this.

The second question also comprised a collection of standard calculations to be completed by hand:

⁴ For the benefit of younger readers, the symbols respectively abbreviate pounds (£), shillings (s) and pence (d). The arithmetic tasks involved are made more complicated and thus more challenging by the relationships that a shilling comprised twelve pence and a pound comprised twenty shillings.

2. (a) Evaluate⁵ $\left(9\frac{1}{2} \times 1\frac{1}{3} \div 7\frac{3}{5}\right)$
 (b) Multiply £6. 18s. 4d. by 47 OR Multiply \$13.83 by 47.
 (c) Divide £508. 7s. 1d. by 65 OR Divide \$1016.60 by 65.
 (d) Simplify $\frac{1.95 \times 7.7 \times .27}{99 \times 19.5 \times .003}$

Note: Absolute accuracy is required.

Students were permitted to complete money calculations in the old or the new currency; it is difficult to imagine students preferring the old over the new, at least as far as the calculations are concerned; while the by-hand calculations are quite complicated for each, they are certainly made a lot easier with decimalisation. It is curious that part (b) offers the same task to students (since there were two dollars to the pound), which seems to exaggerate the difficulty of the older calculation, rather than making the question ‘fair’ either way.

Even at the (much) more sophisticated Leaving Certificate level, students were still expected to complete pure calculations on examinations. For example, the 1955 Leaving Mathematics A examination contained the following part question:

4. (b) Find, correct to two decimal places, the cube root of

$$\frac{(4.36)^4 + (3.12)^4}{7130}$$

Students would need to use logarithms to deal with this question, which is half of one of seven questions in a three-hour paper, so was presumably expected to take around 12 or 13 minutes of exam time. A decade later, on the 1965 paper, students were explicitly told to use logarithms to complete the calculation, but were not advised on the level of accuracy of the result:

1. (a) Use logarithms to evaluate P^2Q^3/\sqrt{R} where $P = 334.2$, $Q = 0.03427$, $R = 10.01$.

This question was expected to consume about 8 minutes of the 140 minute exam, it seems. An innovation in 1965 was that students were permitted to use a slide rule in the examination, but it would not have been appropriate to do so here, except for the purpose of checking the answer, since the number of significant figures exceeds the capacity of a typical slide rule.

It is worth noting at this stage that mathematics students in the 1950s were restricted to using logarithms for complicated calculations, a technology that was around three hundred years old at that stage. Similarly, students in the 1970s were using a slide rule, a means of calculation available for almost as long as logarithms. These were wonderful inventions that significantly extended the working lives of early mathematicians and scientists, since they facilitated calculation to acceptable accuracy for many practical purposes. It is also worth noting that slide rules purchased by students (or more likely their parents) in the late 1960s and the 1970s were not inexpensive items of equipment.

The arrival of the calculator

The calculator was invented late in the 1960s and began to appear in schools in the early 1970s, although at first they were not accessible to individual students (because they were very expensive). Calculations were still expected by students in the strongest mathematics units at the end of

⁵ The two different symbols for fractions, using horizontal and slant vincula respectively were, presumably, used intentionally, although it is also possible that they reflect typesetting limitations of the day. In any event, they are faithfully reproduced here.

secondary school in the 1970s. For example, the 1975 Mathematics II paper included a question which was concerned mostly with completing arithmetical calculations, but by that stage, a recognition that there were several ways of doing so was made explicit:

4. (a) (5 marks)

Evaluate with slide rule (3 figures accuracy) or logarithms

$$(i) \quad \frac{27.3 \times 0.642}{117} \qquad (ii) \quad (8.7)^{\frac{1}{5}} .$$

(b) (2 marks)

Without using tables or slide rule, find

$$(i) \log_5 125 \qquad (ii) \log_{100} 0.1$$

(c) (3 marks)

Use logarithms to compute the value of $\left(1 + \frac{1}{x}\right)^x$ when $x = 100$.

Since the entire paper took three hours, and this was one of thirteen questions to be completed, a question like this was expected to take students about 14 minutes to complete. The conditions for the examination were different from those of a decade earlier, with mathematical tables provided for students (to ensure consistency) and advice about the use of slide rules given on the exam front page: “Candidates using slide rules must show sufficient working to allow their answers to be checked readily.”⁶

Interestingly, there were very few opportunities in the 1975 examination paper for students to undertake calculations related to a *context* of any kind except a pure mathematical context. The singular exception on the entire paper that referred to an everyday context was the following part question:

16. (4 marks)

A desk has a square lid hinged along one edge and inclined at 30° to the horizontal. Find the sine of the angle⁷ which the diagonal of the lid makes with the horizontal. (Leave answer in surd form).

Perhaps ironically, this singular example of mathematics related to an everyday context required that the result be given in exact (i.e. surd) form, which has the effect of extinguishing the sense of everyday reality that might otherwise have been injected into the examination. Arguably, the difficulties associated with calculation at that time had a significant influence on the nature of mathematics (at least as it was manifested in the heady environment of public examinations).

Importantly, the same examination paper also explicitly banned the use of calculators, with the admonition on the cover page: “Electronic calculators may not be taken into the examination room.” Such a warning was necessary since, by the mid-1970s, scientific calculators had become inexpensive enough to be considered for school use, even though they were certainly not regarded

⁶ It is not immediately clear now what this requirement might have entailed for questions like 4(a), however.

⁷ These words were underlined in the examination paper, presumably to ensure that students did not mistakenly find the angle instead of its sine.

as inexpensive⁸ in any absolute sense. The appearance of scientific calculators rendered some of the calculations of school mathematics much easier, and thus provided an opportunity for practical contexts to be given more prominence than had previously been the case.

Not surprisingly, at around that time, members of the MAWA began debating the cases for and against the use of calculators in schools and began exploring ways in which they might influence the mathematics curriculum. An early example was Broderick (1976, p.47), who noted that:

With recent decreases in cost, most students now have access to at least the simplest of the standard calculators. The following examples are possible uses based on the four functions (+, -, ×, ÷) common to all calculators. With sign change (+/-) and memory function now available in models priced at less than \$15 the methods suggested can be easily simplified.

While \$15 does not seem much these days, and indeed there are scientific calculators available these days for around that price, the value of money has changed considerably over the last thirty years. A useful basis for comparison may be the Individual MAWA secondary membership annual fee, which in the same issue of the journal was described as \$8. In other words, a four function calculator might have cost almost twice as much as a year's MAWA membership.

The following year, Colgan (1977, p.37) described in some detail features of scientific and four-function calculators for mathematics teachers to consider, noting along the way some features related to statistics and the costs of calculators with various capabilities:

An increasingly large number of reasonably priced calculators (some around \$30) include some basic statistical functions. The typical case is a machine that provides the mean and standard deviation of a set of numbers. ... A few provide correlation coefficients. Some provide summary data for each set of scores as well. At least one machine around \$40 provides all of the above!

Again, by way of comparison, the MAWA secondary membership fee had increased (dramatically!) to \$12 per year by 1977, so that a scientific calculator with these sorts of capabilities was available for around three times the fee. Such observations suggest that, compared with the cost of calculators these days (including graphics calculators) calculators were relatively expensive at that time.

With declining prices and growing use in schools, it was not surprising that calculators were eventually permitted for use in public examinations, which happened from the 1980 examinations, having been flagged to schools late in 1978. In order to evaluate the likely impact of this change on the mathematics curriculum, and especially on its examination in public examinations, Lange (1979) analysed the 1978 Tertiary Admissions Examination (TAE) for Mathematics II. His analysis concluded with the observations:

The 1978 Mathematics II paper was set before the decision to allow the use of calculators in T.A.E. papers. It is fair to assume therefore that it was set with no attempt to equalize the examination in terms of calculator use or non-use. This is significant, because it is clear from the preceding analysis that the paper would have afforded remarkably little advantage to the calculator user. (p.12)

It is interesting to note that the 1985 Mathematics II examination, some five years after the official sanctioning of calculators in schools contained significantly more questions that involved an everyday context of some kind, and seemed less concerned with being calculator-neutral, as Lange

⁸ I recall that my first scientific calculator, purchased around 1974 and rudimentary by today's standards, cost me about one week's salary as a young teacher.

predicted several years earlier. It is also of interest to note that the instructions for the examination paper still expected students to provide for themselves a (particular) tables book, an approved calculator and (optionally) a slide rule⁹. This is a very substantial change from the means of calculation afforded to students a mere thirty years earlier.

Policy developments

Around Australia, similar developments in access to calculators were part of the reason for an increased focus on what is important about calculation, and what the place of the calculator is within that environment. Major developments in this respect nationally were the publication by the Australian Association of Mathematics Teachers, to which MAWA is affiliated, of agreed policy papers related to calculators. (AAMT 1987, 1996). The first of these offered an unambiguous and nationally agreed position on the place of calculators in schools, suggesting that all pupils, K-12, ought to have adequate access to a calculator whenever they needed it, while the second, in reaffirming this position, acknowledged that calculation also occurs on computers as well as calculators.

Published in between these two policy papers was *A national statement on mathematics for Australian schools* (Australian Education Council, 1991), which also recognised the roles of both calculators and computers in schools, as well as articulating an eloquent case for careful consideration of both mental and paper-and-pencil computation, and a focus on the agency of calculation: that is, students making decisions for themselves about the means of calculation, the desirable levels of accuracy appropriate and the interpretation of results of calculation.

While *A national statement on mathematics for Australian schools* reflected existing curricula, rather than suggesting that there ought to be, or even could be, a thing called a national curriculum in mathematics, it reflected a growing consensus around Australia of the significance of calculation. So it was not surprising that the early versions of identification of outcomes for mathematics, the *National Profiles in Mathematics*, included explicit attention to outcomes associated with calculation. In the case of Western Australia, this document became the forerunner of the identified place of calculation in today's curriculum structures.

Graphics calculators

The recent history of graphics calculators is too fresh to merit much space in an article of this kind. Suffice it to say that, following the early use of graphics calculators in an increasing number of schools in the early 1990s, graphics calculators were permitted for use in senior secondary school examinations in Western Australia from 1996. The initial experiences were similar in many ways to those of scientific calculators, with relatively little impact on examinations themselves, but a rather more substantial impact on the curriculum and the experience of students in school. A major reason for the difference between the two kinds of calculators is that graphics calculators permit students to undertake exploratory work of many kinds relevant to senior school mathematics, while scientific calculators basically permitted only calculations to be undertaken. This view of the significance of the graphics calculator is expanded considerably in Kissane (1995). The increased access to graphics calculators has certainly meant that students had a calculator at their disposal for calculation reasons, even if these are not the most important reasons for using a graphics calculator.

⁹ The continued official acceptance of a slide rule is especially curious, as they had all but disappeared by that time. Indeed, the speed of disappearance of the slide rule from mathematics classrooms rivals that of the disappearance of dinosaurs from planet Earth. The author suspects that there is somewhere a substantial hidden collection of, possibly rusty, slide rules that were made redundant remarkably quickly as scientific calculators arrived in schools.

Calculation in the twenty-first century

As we approach MAWA's fiftieth birthday, and in the light of considerable changes being contemplated to post-compulsory schooling in Western Australia, it is appropriate to consider briefly some recent changes in calculation available to secondary students and ponder their significance. It is important to consider such changes in the light of the curriculum structures within which they operate. Significantly, the closing years of the twentieth century and the opening years of the twenty-first century have seen a more careful identification of the outcomes of schooling, including those associated with calculation. Unlike earlier times, we now have an explicit outcome that identifies the significance of calculation within the mathematics curriculum and, indeed, there is an outcome in the Curriculum Framework (Curriculum Council of WA, 1998, p. xx) named *Calculate*:

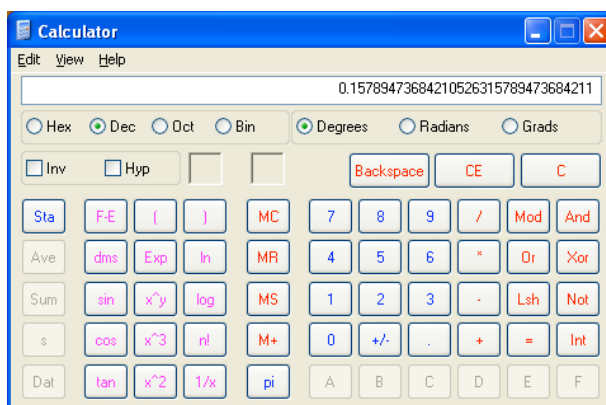
Calculate

Students choose and use a range of mental, paper and calculator computational strategies for each operation, meeting needed degrees of accuracy and judging the reasonableness of results.

Identifying calculation as an outcome provides a clear expectation that it ought to an extent be paid careful attention in instruction and that we ought to be explicit about what it means to get better at it. While we have always known it was important, it is only recent developments of this kind that have allowed us to sharpen our focus on what it is about calculation that is important. The most effective means of making this clear are the Progress Maps, recently published by the Curriculum Council, identifying how attainment of this outcome grows developmentally.

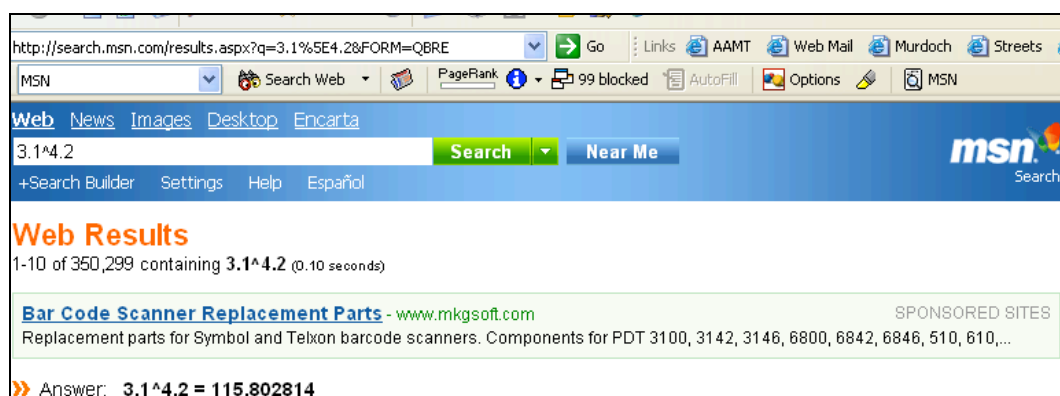
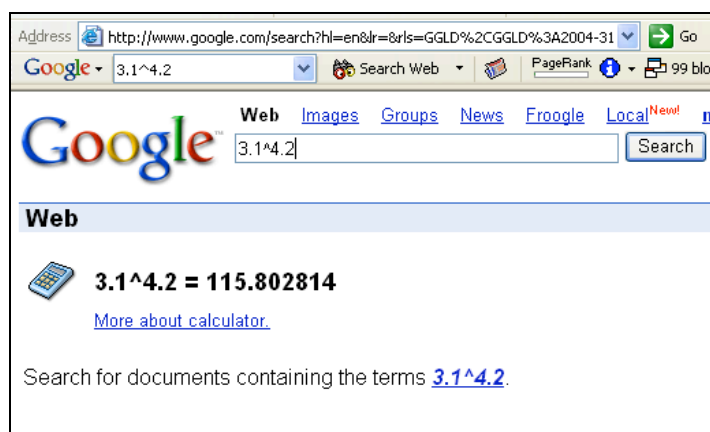
Recent trends in access to technology have again changed the landscape considerably. In a paper that is already too long, there is place for only a few examples of the changing landscape.

One change involves the widespread access to computer technology, both in schools and at home. Inevitably, this will provide students with access to calculation tools not previously accessible. As one example, the screen below shows the calculator that comes bundled with the Windows XP operating system, in widespread common use. Of interest is the fact that the calculator display offers very many more digits of accuracy than is the case for typical hand-held calculators. The calculation shown ($3 \div 19$) would normally produce only a few decimal places, so that the repeating structure of eighteen digits is visible to the user.



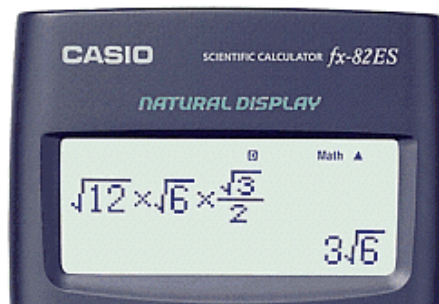
As a second example of the influence of computers, one of the remarkable changes of recent times is the invention and rapid penetration of the Internet into western countries (at least), and recently into a large and rapidly growing proportion of Australian households. Indeed, it seems that many secondary students today have better effective access to the Internet at home than they do at school. That being the case, it is of interest to note that Search engines provide access to calculation online,

merely by typing in a calculation into the search window The screen below shows an example from the two Internet rivals, Google and MSN:



In this case, the results obtained are no better than those accessible on a typical scientific calculator, and (characteristic of the web), the result for MSN also offers a complimentary advertisement instead of a result for the calculation. Nonetheless, this is a development worth noting, especially if the proprietors of such facilities choose to increase the range of capabilities on offer, available to people whenever they are using a browser¹⁰.

Hand-held technologies seem likely to continue to develop, and provide students and others with ready access to calculation capabilities. A recent calculator, for example, illustrated below has pushed the boundaries of calculation a little wider by offering some exact calculation capabilities¹¹, in addition to decimal capabilities.

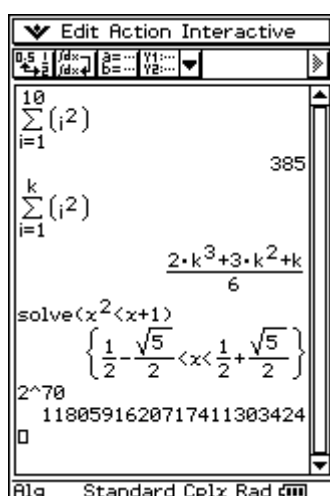


¹⁰ It may also be worth observing, at the same time, that most mobile phones these days also have an inbuilt calculator. At present, these too are fairly primitive (even more so than the browsers), but the urge to provide more facilities than the competitors may lead some manufacturers to develop more sophisticated versions. There would seem to be no other impediment to doing so, as the technology already contains substantial processing power.

¹¹ Interestingly, the same calculator provides exact values for trigonometric ratios of angles that are multiples of 15°.

Previously, inexpensive technologies have offered only numerical approximations to calculations of the kind shown here. This is no longer the case, as shown by the illustration above of a calculator costing around \$25 of 2005 money. Continuing the benchmark comparisons with the cost of MAWA secondary membership, this is less than one third of the annual membership fee (\$84), thus representing a change by a factor of around nine since Colgan's observations back in 1977. The scientific calculator has decreased in price by a factor of about nine, while increased in capabilities quite substantially over the past quarter of a century.

Finally, recent hand-held technologies such as Casio's *ClassPad 300* offer very significantly calculation capabilities to users, who are increasingly likely to be secondary school students. The screen below shows a few examples of what have now become routine calculations in such an environment, and suggest a continuing blurring of the boundary between calculation and analysis.



To use the MAWA membership comparison one last time, it is a good measure of the rate of change of access to means of calculation that the price of a *ClassPad 300* today seems to be a little more than twice the annual secondary membership fee. This was the sort of ratio referred to above by Broderick (1976) for the case of a four-function calculator around 30 years ago¹². By any measure, this is an extraordinary change.

Conclusion

It is clear that there has been a breathtaking change in calculation capabilities over the comparatively brief lifespan of our professional association. It is not easy, in the midst of such changes, to see the bigger picture, but it is important to try to do so. Tools available everyday now to students and citizens at large can handle efficiently and quickly the calculations expected at the end of school fifty years ago.

While calculation will continue to be an important part of mathematics (and people's perceptions of what it is), and so will continue to be an important part of what we try to help students develop confidence and competence with in secondary school, the nature of calculation fifty years after the founding of our association is remarkably different from what it was at the beginning. While calculation did not change much in mathematics itself for a few hundred years up to the foundation of MAWA, it has changed beyond recognition since that time. Charting an intelligent course through the increasing range of possibilities is not easy, but will require members of MAWA to keep abreast of the changes and the wider societal circumstances in which calculation is important.

¹² Although the arithmetic is not precisely correct, it seems plausible that it will be about right by the time the figure of 30 is correct in around a year or so.

Acknowledgement

I am grateful to the Curriculum Council of Western Australia for providing copies of old examination papers and for granting permission to reproduce some questions. I am also grateful to the Mathematical Association of Western Australia office staff, especially to Bobbi McMullan, for providing old copies of *Sigma*, the MAWA's journal for many years.

References

- Australian Association of Mathematics Teachers 1987. *A national statement on the use of calculators for mathematics in Australian schools*. Canberra: Curriculum Development Centre.
- Australian Association of Mathematics Teachers 1996. *Statement on the use of calculators and computers for mathematics in Australian schools*. Adelaide: Author.
- Australian Education Council 1991. *A national statement on mathematics for Australian schools*. Carlton, Victoria: Curriculum Corporation.
- Blakers, L. 1978 M.A.W.A. – An historical note. *Sigma*, 100, 21-23.
- Broderick, B. 1976. Algorithms and the mini-calculator. *Sigma*, 91, 47-52.
- Colgan, J. 1977. Calculator characteristics. *Sigma*, 94, 26-39.
- Curriculum Council of Western Australia 1998. *Curriculum Framework*. Osborne Park: Author.
- Kissane, B. 1995. The importance of being accessible: The graphics calculator in mathematics education. *Proceedings of the First Asian Technology Conference in Mathematics*, Association of Mathematics Educators: Singapore, 161-170.
- Lange, B. 1979. Calculators and the T.A.E. *Sigma*, 103, 1-15.
- University of Western Australia 1955. *Junior Certificate Examination: Arithmetic and Algebra*. Perth: Author.
- University of Western Australia 1955. *Leaving Certificate Examination: Mathematics A*. Perth: Author.
- University of Western Australia 1965. *Junior Certificate Examination: Arithmetic and Algebra*. Perth: Author.
- University of Western Australia 1965. *Leaving Certificate Examination: Mathematics A*. Perth: Author.
- Tertiary Institutions Service Centre 1975. *Tertiary Admissions Examination: Mathematics II*. Perth, Author.

* Paper is published as:

Kissane, Barry 2006, Half a century of calculation in Western Australian secondary schools. *Cross Section*, 16(1) 1-11.